# The axiom of covering homotopy for locally flat embeddings

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The space of locally flat embeddings of a compact manifold $M$ in a closed manifold $N$ is considered. Let some locally flat embedding $q^*$ be fixed, and $M$ be identified with $q^*M$. Let $\mathfrak{E}$ be the space of all such locally flat embeddings $q : M \to N$ which may be connect with $q^*$ by isotopy $q_t$, where all $q_t$ are locally flat. $\mathfrak{H}$ will designate the identity component of the group of homeomorphisms of $N$ onto itself in the uniform topology. At last, let the epimorphic mapping $p : \mathfrak{H} \to \mathfrak{E}$ be defined by equality $p(h) = hq^*$.

**Theorem 1.** *If* $\dim N - \dim M \neq 2$ *and* $dimN \geq 5$ *the mapping* $p$ *is Serre fibering.*

An outline of a proof was published in German in Hausdorf memory volume (Theory of sets and topology, Berlin 1972, p. 503-508). A full exposition with some complements is now prepared.

The proof is based on the fundamental theorem of E. Michael [1]:

**Theorem M**. *Let* $X$ *be a paracompact space,* $\dim X \leq n$*,* $A$ *a closed subset of* $X$*,* $Y$ *a complete metric space, and* $f : Y \to X$ *such a mapping that the corresponding decomposition of* $Y$ *is low semicontinuous and is "equi* $n$*-LC". Then every section over* $A$ *has a continuation over some neighborhood of* $A$*.*

(Michael called a fibration f " equi $n$-LC ", if for any point $y$ of every fibre $f^{-1}x$ and for any $\varepsilon > 0$ there is a $\delta > 0$, such that arbitrary mapping of $k$-sphere ($k \leq n$) into $O_\delta(y) \cap f^{-1}x',\ x' \in X$ is contractible to point through $O_\varepsilon(y) \cap f^{-1}x'$.)

Our application of Michael theorem relies on two theorems about homeomorphism group and locally flat embeddings.

**Theorem O.** *Subgroup of the group* $\mathfrak{H}$*, consisting of homeomorphisms fixed pointwise on a locally flat submanifold* $M \subset N$ *is locally contractible.*

A proof is given by the author in [2].

**Theorem A.** *If* $\dim N - \dim M \neq 2$ *and* $dimN \geq 5$*, for any* $\varepsilon > 0$ *there exists a* $\delta > 0$*, such that for every locally flat embedding,* $\delta$*-close to* $q^*$*, there is an isotopy which transfers* $q$ *in* $q^*$*.*

The proof of this result uses the same technique ("infinite repetitions"), as the theorem **O**, but also relies on Briant – Seebeck theorem [3], whose proof demands a variant of *Engulfing lemma*, and this leads to the dimension condition. The theorem **A** shows that sufficiently small neighborhood in $\mathfrak{E}$ is an image of a small (contractible through bigger one) neighborhood in $\mathfrak{H}$.

With help of theorem 1 and local contractibility of the homeomorphism group one obtains the local $n$-connectivity of the space $\mathfrak{E}$:

**Theorem 2.** For any $n$ and every neighborhood $U$ of every point $q \in \mathfrak{E}$ there is a smaller neighborhood $V$ such that any mapping of the $n$-sphere in $V$ is contractible to a point through $U$.

The proof consists in two steps: the mapping of sphere firstly lifts to a contractible neighborhood in $\mathfrak{H}$, and then the projection $p$ transfers the contraction of the lifted mapping into the demanded contraction of the given mapping.

## References

[1] E. MICHAEL, Continuous selections II. Ann. Math. 64 (1956), 562 — 580.

[2] А.В. ЧЕРНАВСКИЙ, Мат. сб. 79 (1969), 307 – 366.

[3] А.В. ЧЕРНАВСКИЙ, ДАН СССР 187 (1969), 1247-1250.

[4] J.L. BRYANT and C.L. SEEBECK, Bull. Amer. Math. Soc. 70 (1968), 378-381.