ON UNIQUE SOLVABILITY OF SHOWALTER PROBLEM FOR A DEGENERATE EVOLUTION EQUATION

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Various initial boundary value problem for partial differential equations or systems of equations can be investigated in the framework of the Showalter problem

$$Pu(t_0) = u_0, \tag{1}$$

$$L\,\dot{u}(t) = Mu(t) + N(t, u(t)), \quad t \in [t_0, T]. \tag{2}$$

Here $\mathfrak{U}$ and $\mathfrak{F}$ are Banach spaces, operator $L : \mathfrak{U} \to \mathfrak{F}$ is linear and continuous, $\ker L \neq \{0\}$, $M : \mathrm{dom}M \to \mathfrak{F}$ is linear, closed and densely defined in $\mathfrak{U}$, operator $N : [t_0, T] \times \mathfrak{U} \to \mathfrak{F}$ is nonlinear.

Definition 1. *Strong solution* of problem (1), (2) is a function $u \in H^1(t_0, T; \mathfrak{U})$, such that conditions (1) and (2) are fulfilled for almost all $t \in [t_0, T]$.

If operator M is strongly (L, p)-radial then the spaces $\mathfrak{U}$ and $\mathfrak{F}$ are direct sums $\mathfrak{U} = \mathfrak{U}^0 \oplus \mathfrak{U}^1$, $\mathfrak{F} = \mathfrak{F}^0 \oplus \mathfrak{F}^1$ [1]. Denote $M_1 = M|_{\mathfrak{U}^1 \cap \mathrm{dom}M}$.

Theorem 1. *Let $\mathfrak{U}$ be a reflexive Banach space, operator M be strongly (L, p)-radial, operator $N : [t_0, T] \times \mathfrak{U} \to \mathfrak{F}$ be Lipschitzian with respect to two variables, $\mathrm{im}N \subset \mathfrak{F}^1$. Then for every $u_0 \in \mathrm{dom}M_1$ problem (1), (2) has a unique strong solution on $[t_0, T]$.*

Example. Let $\Omega \subset \mathbb{R}^n$ be a bounded region with a boundary $\partial\Omega$ of the class C^∞, $n < 4$, $g : \mathbb{R} \times \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$. Find a function $z = z(x, t)$, defined in $\overline{\Omega} \times [t_0, T]$, such that

$$(\lambda - \Delta) z_t(x, t) = \alpha \Delta z(x, t) - \beta \Delta^2 z(x, t) + (\lambda - \Delta) g(t, x, z(x, t)), \ (x, t) \in \Omega \times [t_0, T],$$

$$z(x, t) = 0, \ (x, t) \in \partial\Omega \times [t_0, T],$$

$$(\lambda - \Delta) z(x, t_0) = z_0(x), \ x \in \Omega.$$

This equation models a free surface evolution for a filtered fluid [2]. Here $\lambda, \alpha \in \mathbb{R}$, $\beta \in \mathbb{R}_+$ are parameters that define the fluid.

Список литературы

[1] G. A. Sviridyuk, V. E. Fedorov. *Linear Sobolev Type Equations and Degenerate Semigroups of Operators*, VSP, (2003).

[2] E. S. Dzektser, «Generalization of the motion equation for ground water witha free surface», *Doklady AN SSSR*, Vol. 202, No. 5, 1031—1033 (1972).

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