LEGENDRIAN KNOTS, MONOTONIC SIMPLIFICATION, AND JONES' CONJECTURE ON BRAIDS

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It is shown in [1] that every rectangular (or grid) diagram of the unknot admits a monotonic simplification to the trivial rectangular diagram, which is just a square. A monotonic simplification means a finite sequence of elementary moves that do not increase the number of edges of the diagram.

In a recent joint work with M.Prasolov [2] we found a necessary and sufficient condition for a rectangular diagram D of a link to admit a monotonic simplification to a diagram with a smaller number of edges. The condition is expressed in terms of two Legendrian links $L_1(D)$ and $L_2(D)$ naturally associated to the diagram D:

Theorem. *A rectangular diagram D admits a non-trivial monotonic simplification if and only if at least one of $L_1(D)$ and $L_2(D)$ admits a Legendrian destabilization.*

This result (in a slightly stronger form, which we skip for brevity) has a number of corollaries among which the following one, which is known as Jones' conjecture:

Theorem. *Let β_1, β_2 be two braids such that their closures are equivalent as oriented links and their braid index is minimum possible for the corresponding link type. Then β_1 and β_2 have the same algebraic crossing number.*

References

[1] I.Dynnikov, "Arc-presentations of links: monotonic simplification", *Fund. Math.*, Vol. 190 (2006), 29–76

[2] I.Dynnikov, M.Prasolov, "Bypasses for rectangular diagrams. Proof of Jones' conjecture and related questions" (Russian), *Trudy Moskovskogo Matematicheskogo Obschestva*, Vol. 74, No. 1 (2013), 115–173; translation in *Trans. Moscow Math. Soc.*, 2013, pp. 97–144.

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