HYPERBOLIC MANIFOLDS WHICH BOUND GEOMETRICALLY

LAPPAROV VALENTIN

The following question was considered by many authors: which compact orientable hyperbolic n-manifold N can represent the totally geodesic boundary of a compact orientable hyperbolic $(n+1)$-manifold M? Once there exist such manifolds N and M, we say that N *bounds* M *geometrically*.

In [1] Kolpakov, Martelli and Tschantz constructed an explicit infinite family in dimension $n = 3$ using the right-angled dodecahedron D and 120-cell Z. These two compact Coxeter right-angled regular polytopes exist in $\mathbb{H}^3$ and $\mathbb{H}^4$ respectively, and the first is a facet of the second. There volumes are known: $\mathrm{vol}(D) = 4.3062\ldots$ and $\mathrm{vol}(Z) = \frac{34}{3}\pi^2$.

The construction of manifolds is based on assigning appropriate colourings to facets of D and Z following Davis and Januszkiewicz [2], as well as Izmestiev [3] and Vesnin [4, 5].

The following resilt was obtained in [1]. For every $n > 1$ there exists an orientable compact hyperbolic three-manifold N_n of volume $16n\,\mathrm{vol}(D)$ which bounds geometrically an orientable compact hyperbolic four-manifold M_n of volume $32n\,\mathrm{vol}(Z)$. The manifolds N_n and M_n are tessellated respectively by $16n$ right-angled dodecahedra and $32n$ right-angled 120-cells. The key observation is that a colouring of the dodecahedron D can be enhanced in a suitable way to a colouring of the right-angled hyperbolic Coxeter 120-cell Z. To get infinitely many examples n copies of D and Z can be taken.

In our report, we will discuss this issue further generalizations.

References

[1] A. Kolpakov, B. Martelli, S. Tschantz, "Some hyperbolic three-manifolds that bound geometrically" arXiv:1311.2993, to appear in *Proc. Amer. Math. Soc.*

[2] M. Davis, T. Januszkiewicz, "Convex polytopes, Coxeter orbifolds and torus actions" *Duke Math. J.* 62 (1991), 417–451.

[3] I. Izmestiev, "Three-dimensional manifolds defined by coloring a simple polytope" *Math. Notes* 69 (2001), 340–346.

[4] A. Vesnin, "Three-dimensional hyperbolic manifolds of Loebell type" *Siberian Math. J.* 28 (1987), 731–733.

[5] A. Vesnin, "Three-dimensional hyperbolic manifolds with a common fundamental polyhedron" *Math. Notes* 49 (1991), 575–577.

Novosibirsk State University, Pirogova Str, 2, Novosibirsk, 630090, Russia
E-mail address: vlapparov@mail.ru

The research is supported by the RFBR grant number 13-01-00513.