PARITY, PATTERNS, AND GRAPH-VALUED QUANTUM INVARIANTS IN LOW-DIMENSIONAL TOPOLOGY

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Usually, invariants of knots and other objects in topology are value in some algebraic objects like *modules*, *groups*, *polynomial rings* etc.

Looking at the value of the invariant, one can judge about the knot quite implicitly. For example, the value of polynomials and quantum invariants of knots usually can give some estimates on the crossing number, genus, and other characteristics of knots, but looking at the value of an invariant, one can not say too much about the form of the knot.

The aim of the present talk is to prove various versions of the following

Theorem. *If a (virtual) knot diagram K is complicated enough then it is contained in every diagram K' equivalent to it.*

View Fig. 1.

This theorem was first proved in [2] in the form that every *irreducibly odd* diagram (of a virtual knot) is contained in every diagram K' equivalent to it.

Here a *parity* is an axiomatic property of crossings of the diagram which behave nicely under Reidemeister moves, see [2]. So, for every *parity* in every knot theory, there are various classes of diagrams K which are minimal in this strong sense.

The key observation to the proof of this theorem is that there exist invariants of virtual knots $K \to [K]$ such that if a knot diagram K is complicated enough then

$$[K] = K.$$

In the above equality, K in the left hand side means the knot K (the equivalence class) whose invariant is considered, and K in the right hand side means the diagram K. So, if K' is equivalent to K then $[K'] = K$ which means by construction that K' contains K as a subdiagrams.

It turns out that the same idea can be realized by using formalisms due to Kuperberg, [1]. G.Kuperberg suggested a way for evaluating quantum A_2, C_2, and G_2 for quantum invariants

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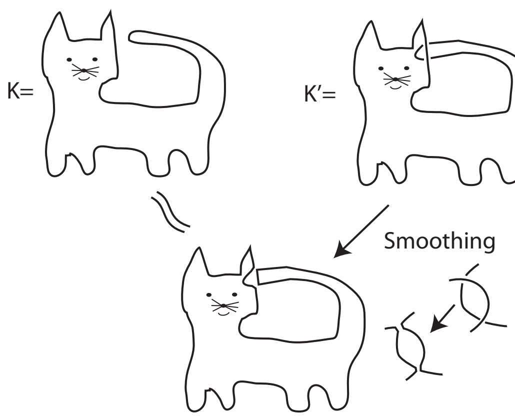

FIGURE 1. A diagram K is a subdiagram of a diagram K' equivalent to it

by expanding (classical) knot diagrams as linear combinations of graphs, which, in turn have to be considered modulo some *reductive moves*.

For classical knots, such graphs are simple enough, so that the invariants evaluate to *polynomials*. In the case of virtual knots graphs may be complicated enough so that they *get evaluated to themselves*. [See [3, 4] for A_2 and C_2, the G_2 case is a joint paper of the author and L.H.Kauffman in preparation.]

Free Links (a simplifiaction of virtual links) were conjectured to be trivial by Turaev [5] in 2004.

Since the first use of parity by the author in 2009 to disprove Turaev's conjecture, various variations and enhancements of **the Theorem** were used for constucting invariants of cobordisms of free knots, construction of functorial mappings in various knot theories, in particular, constructing the functorial Gauß diagram mapping from virtual knots to classical knots, reducing questions about (virtual) knots to questions about their representatives, almost complete classification of free links, applying the parity techniques to classical knots.

References

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