QUANTUM $\mathcal{R}$-MATRICES AND INTEGRABLE SYSTEMS

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We will discuss the connections between the quantum $\mathcal{R}$-matrices and integrable systems, in particular with τ-functions of KP/Toda-equation.

Using the Reshetikhin-Turaev formalism the HOMFLY polynomials $H^{\mathcal{K}}$ of the knot can be described as a trace over product of quantum $\mathcal{R}$-matrices. This procedure provides a character expansion of the HOMFLY polynomial from the braid. For HOMFLY polynomials characters should be taken in some particular points called topological locus. These expressions then can be generalized to the case of characters taken in arbitrary points. The resulting polynomials, called generalized HOMFLY polynomials $\mathcal{H}^{\mathcal{K}}$, are no longer topological invariants but only braid invariants.

$$H_R^{\mathcal{K}} = \sum_Q h_R^Q S_Q(A,q) \qquad\qquad p_k^* = \frac{A^k - A^{-k}}{q-q^{-1}}$$

$$\mathcal{H}_R\{p\} = \sum_Q h_R^Q S_Q\{p\} \tag{1}$$

For a special class of knots called torus knots these generalized HOMFLY polynomials satisfy Plucker relations and therefore are τ-functions of KP/Toda-equations. Generalized HOMFLY polynomials can be constructed not only for torus knots but for any knots, but it remains to understand what is the corresponding analog of τ-functions.

References

[1] A.Mironov, A.Morozov and And.Morozov, "Character expansion for HOMFLY polynomials I: Integrability and difference equations", Contribution to the *Strings, Gauge Fields, and the Geometry Behind*, 2012.

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