TRANSFINITE MILNOR INVARIANTS FOR 3-MANIFOLDS

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In collaboration with Jae Choon Cha, of POSTECH University, we extend the definition of Milnor's invariants from links to oriented, closed 3-manifolds, and show that in this setting transfinite Milnor invariants of 3-manifolds can be realized.

In his early work, John Milnor defined invariants of links which measure the longitudes of the link relative the lower central series of the link group. Stallings theorem implies these are invariants of the topological concordance class of the link. Among his remaining unanswered questions, Milnor asked how one can extract invariants from the intersection of the lower central series of the group, that is, how can one define *transfinite* versions of Milnor's invariants?

Unlike the classical lower central series, Jae Choon Cha and I showed that the transfinite lower central series quotients of a link group are not invariant under concordance [1]. Nonetheless, candidates for non-zero transfinite Milnor invariants exist.

I defined the first transfinite Milnor invariant in [5], which was later significantly improved by J. P. Levine by introducing his *algebraic closure of groups* [3, 4]. The general problem remains open, as efforts have, so far, failed to realize the above invariants.

In this talk, we present extensions of Milnor's link invariants to closed oriented 3-manifolds, and show that in this broader setting, there are distinct three manifolds with the same finite Milnor invariants, but which can be distinguished via transfinite Milnor invariants. Additionally, we give geometric interpretations for the vanishing of Milnor invariants for 3-manifolds, extending known interpretations for the vanishing of classical Milnor's invariants of links [2, 5].

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