TOPOLOGICAL QUANTUM COMPUTATION

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Quantum computation is any model for computation that requires the ability to create, manipulate and measure quantum states. Topological quantum computers (TQC) will be built upon physical systems with topological degrees of freedom, called topological phases. These topological degrees of freedom confer an inherent fault-tolerence on the computational model, obviating the need for the costly quantum error-correction that plagues the quantum circuit model.

A quantum mechanical system is in topological phase if the effective field theory for the system (at long wavelengths and low energies) is a Topological Quantum Field Theory (TQFT). Topological phases of matter are of great interest themselves: examples include fractional quantum Hall liquids and spin liquids studied in condensed matter physics.

In the early 1990s Turaev defined modular categories (following earlier ideas of Moore and Seiberg in rational conformal field theory) and showed that they lead to (2+1)TQFTs. Indeed, it is expected that the two concepts are essentially equivalent. For this reason, we may regard modular categories as the programming language of TQC, and as the symmetries of topological phases of matter. Fundamental questions that arise in physics and quantum computation can therefore be addressed mathematically by studying modular categories.

In two lectures I will discuss:

(1) How topological quantum computers work.
(2) The concept of a modular category and its relationship with TQC.
(3) Translations of problems in TQC into algebraic questions.
(4) Some recent progress on these problems.

In particular I will highlight:

The recent resolution of the rank-finiteness conjecture for modular categories obtained with Bruillard, Ng and Wang [1]:

Theorem. *There are finitely many modular categories of any fixed rank*

The evidence obtained with several coauthors including Etingof, Witherspoon [2], Naidu [3] and Wenzl [4] for:

Conjecture. *The braid group representations associated with a modular category $\mathcal{C}$ have finite image if and only if $\mathcal{C}$ is weakly integral.*

All concepts will be defined in the talks.

References

[1] P. Bruillard, S.-H. Ng, E. Rowell and Z. Wang, "On Modular Categories," preprint: arXiv:1310.7050.
[2] P. Etingof, E. Rowell and S. Witherspoon, "Braid representations from twisted quantum doubles of finite groups", **Pacific J. Math**. 234 no. 1 (2008) 33-42.
[3] D. Naidu and E. Rowell, "A finiteness property for braided fusion categories" **Algebr. Represent. Theory**, 15 (2011) no. 5, 837-855.
[4] E. Rowell and H. Wenzl, "$SO(N)_2$ Braid Group Representations are Gaussian" preprint: arXiv:1401.5329.

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