THE EXTENDED COMPLEXITY OF SOME 3-MANIFOLDS

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In [3] was constructed the extended complexity $\overline{c}(M)$ of a orientable irreducible 3-manifold M. $\overline{c}(M)$ represents ordered tuple of five integers, two of which are determined by almost simple spine of manifold [2], and three remained characteristics of the manifold.

The extended complexity has two important properties.

Properties 1. *The extended complexity always decreases when a manifold is cut along an incompressible surface.*

Properties 2. *For any tuple of five integers* $(k_1, k_2, k_3, k_4, k_5)$ *there exists only a finite number of distinct compact orientable irreducible 3-manifolds that have extended complexity* $(k_1, k_2, k_3, k_4, k_5)$.

In this work we consider extended complexity of some series of orientable irreducible 3-manifolds.

Example 1. S^3, D^3, RP^3 *has the extended complexity* $(0, 0, 0, 0, 0)$.

Example 2. *Let* M *be a handle body of genus* g. *Then* $\overline{c}(M) = (0, 0, 0, 0, g^2)$.

Example 3. *Let* $M = F \times I$, F *is a closed surface. If* F *is a orientable surface of genus* g, *then* $\overline{c}(M) = (0, 0, 0, 1, 2g^2)$. *If* $F = \#nRP^2$, *then* $\overline{c}(M) = (0, 0, 0, 2, (n-1)^2)$.

Theorem 1. *If the extended complexity of a connected irreducible manifold* M *is equal* $(0, 0, 0, k_4, k_5)$, $k_4, k_5 \in \mathbb{N}$, *then* M *is the boundary connected sum of the manifolds described in Ex.1–3.*

Theorem 2. *Let* $M = F \times S^1$, F *is a sphere with* k *holes* $(k \geq 3)$. *Then* $\overline{c}(M) = (0, 2k-4, -k, 0, k)$.

Theorem 3. *Let* $M = F \times S^1$, F *is a orientable surface of genus* g *with* k *holes* $(g \geq 1, k \geq 1)$. *Then* $\overline{c}(M) = (0, 4g + 2k - 4, -k, 0, k)$.

References

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