ON THE NORMAL FORM OF KNOTS

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The talk is a report on the results of computer experiments with an algorithm that takes classical knots to what we call "normal form" (and so can be used to identify the knot type of a knot from its knot diagram). We define the *normal form* of a knot as the curve in 3-space corresponding the global minimum of a certain functional.

The algorithm minimizes the functional defined as the sum of (a discrete version of) the Euler functional and a repulsive functional (which forbids self intersections and crossing changes). The Euler functional is

$$E(\gamma) = \int_0^{2\pi} \big(\kappa(\gamma(s))\big)^2 ds,$$

where $\gamma : \mathbb{S}^1 \to \mathbb{R}^2$ is a curve of length 2π (defining the knot), s is the arclength parameter, and $\kappa(\gamma(s)$ is the curvature of γ at the point s. The repulsive functional that we use is very simple: it is zero except when points that are not too close on the curve become too close in space; in that case it becomes huge.

The algorithm is a kind of (discrete) gradient descent along our functional.

We will describe the algorithm and demonstrate how it works; present the knot table of normal forms of prime knots with seven crossings; observe that the algorithm does not necessarily bring knots to normal form (it often stabilizes at local minima which are not global); see that it (unfortunately) fails to untangle some knot diagrams of the trivial knot; note that (fortunately) in all the experiments performed (there were thousands) there were no occurrences of isotopic knots with different normal forms and no nonisotopic knots with the same normal form.

A few physical experiments with wire knots [1] will also be demonstrated; they show a remarkable coincidence with the computer experiments, so we can say that our functional has a natural physical meaning.

The results are joint work with Sergey Avvakumov and Oleg Karpenkov [2].

References

[1] A.B.Sossinsky, "Normal forms of twisted wire knots." Russian J. Math. Physics Vol.19, no.3, 394-400, 2012.

[2] S.Avvakumov, O.Karpenkov, A.Sossinsky, "Euler elastics in the plane and the Whitney-Graustein theorem," Russian J. Math. Physics, Vol.20, no.3, 257-267, 2013.

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