LOCAL SET OF COHOMOLOGY IN PROBLEMS OF ANALYTIC CLASSIFICATION.

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In [1] there was marked relation between a set of one-dimensional cohomology and a space of moduli in problems of analytic classification. Here we establish an analogous result for one class of two-dimensional resonant germs of maps.

Let $v_0 = -x\frac{\partial}{\partial x} + y(1+xy)\frac{\partial}{\partial y}$, and $F_0 = g^1_{v_0}$ be a one-time map of the vector field v_0. Recall that germs F and G are analytically equivalent, $F \sim G$, if one of them can be transformed into another by a local holomorphic change of variables.

Let $\mathcal{F}$ be a class of formal equivalency of the germ F_0. Denote by X the factor-space of direct product of two punctured discs $(\mathbf{C}_*, 0) \times (\mathbf{C}_*, 0)$ by the action of F_0. Let $\mathcal{H}^1(X)$ be the set of local one-dimensional cohomology with values in the sheaf of germs of two-dimensional holomorphisms, defined in correspondence with [1].

Theorem. $\mathcal{F}/\sim = \mathcal{H}^1(X)$.

Remark 1. This theorem says that there exists one-to-one correspondence between classes of analytic equivalency of germs of $\mathcal{F}$ and elements of $\mathcal{H}^1(X)$. We can show that this correspondence is analytic in a some natural sense.

Remark 2. Simple description of the set $\mathcal{H}^1(X)$ can be obtained by the following way. Let us consider the class $\mathcal{M}$ consisting of collections $(\Phi_\pm, \Psi_\pm, \phi_\pm, \psi_\pm)$, where $\Phi_\pm$ and $\Psi_\pm$ ($\phi_\pm$ and $\psi_\pm$) are germs of holomorphic functions in $(\mathbf{C}^2, 0)$ (correspondently, in $(\mathbf{C}, 0)$). Denote by M the space of orbits of the action on $\mathcal{M}$ of the group of linear transformations. Then $\mathcal{H}^1(X)$ may be identified with M.

Remark 3. Using the last identification, we can show that a germ $F \in \mathcal{F}_0$ is embeddable in a flow (i.e. F is 1-time shift for a holomorphic vector field v) iff the components $\Phi_\pm, \Psi_\pm$ of its moduli vanish. Moreover then the pair $\{\phi_\pm\}$ is a Martinet-Ramis moduli (of orbital classification) [2] of the corresponding vector field v, and the pair $\{\psi_\pm\}$ is an additional moduli (of non-orbital classification) [3] of v.

References

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