DISTANCE-REGULAR GRAPHS WITH STRONGLY REGULAR LOCAL SUBGRAPHS HAVING EIGENVALUE 4

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We consider undirected graphs without loops or multiple edges. For a vertex a of a graph Γ the subgraph $\Gamma_i(a) = \{b \mid d(a,b) = i\}$ is called i-neighbourhood of a in Γ. We set $[a] = \Gamma_1(a)$, $a^\perp = \{a\} \cup [a]$.

The degree of a vertex a of Γ is the number of vertices in $[a]$. A local graph of Γ is the subgraph induced by $[x]$ for a vertex x of Γ. A graph is called regular of degree k, if the degree of any its vertex is equal to k. The graph Γ is called amply regular with parameters (v,k,λ,μ) if Γ is regular of degree k on v vertices, and $|[u] \cap [w]|$ is equal to λ, if u adjacent to w, and is equal to μ, if $d(u,w) = 2$. An amply regular graph of diameter 2 is called strongly regular.

If $d(u,w) = i$ then by $b_i(u,w)$ (by $c_i(u,w)$) we denote the number of vertices in $\Gamma_{i+1}(u) \cap [b]$ (in $\Gamma_{i-1}(u) \cap [b]$). The graph Γ with diameter d is called distance-regular with intersection array $\{b_0, b_1, ..., b_{d-1}; c_1, ..., c_d\}$ if $b_i = b_i(u,w)$ and $c_i = c_i(u,w)$ for every $i \in \{0, ..., d\}$ and for every vertices u, w with $d(u,w) = i$. Distance-regular with diameter 2 is called strongly regular with parameters (v,k,λ,μ), where v is the number of vertices of the graph, $k = b_0$, $\lambda = k - b_1 - 1$ and $\mu = c_2$.

A partial geometry $pG_\alpha(s,t)$ is a geometry of points and lines such that every line has exactly $s+1$ points, every point is on $t+1$ lines (with $s > 0,\ t > 0$) and for any antiflag (P,y) there are exactly α lines z_i containing P and intersecting y. In the case $\alpha = 1$ we have generalized quadrangle $GQ(s,t)$.

The point graph of a partial geometry $pG_\alpha(s,t)$ has points as vertices and two points are adjacent if they are incident to the same line. The point graph of a partial geometry $pG_\alpha(s,t)$ is strongly regular with parameters $v = (s+1)(1+st/\alpha)$, $k = s(t+1)$, $\lambda = s-1+(\alpha-1)t$, $\mu = \alpha(t+1)$. A strongly regular graph with these parameters for some natural numbers s,t,α is called pseudo-geometric graph for $pG_\alpha(s,t)$.

J. Koolen suggested the program of investigations of distance-regular graphs whose local graphs are strongly regular graphs with the second eigenvalue at most t for some natural t. Recently this program was completed for $t = 3$ (Makhnev A., Paduchikh D. and others).

Let Γ be a distance-regular graphs whose local graphs are strongly regular graphs with the second eigenvalue r, $3 < r \leq 4$, u is the vertex of Γ and $\Delta = [u]$. Then one of the following holds:

(1) Δ is a pseudo-geometric graph for $pG_t(t+4,t)$;

(2) Δ is the complement graph of a pseudo-geometric graph for $pG_5(s,4)$;

(3) Δ is a union of isolated 5-cliques;

(4) Δ is a conference graph with parameters $(4n+1, 2n, n-1, n)$;

(5) Δ belongs to the finite set of exceptional strongly regular graphs with nonprincipal eigenvalue 4.

In [1] it is classified graphs with local subgraphs from (1-2), (4).

In [2] A. Makhnev and D. Paduchikh obtained parameters of pseudogeometric exceptional graphs with nonprincipal eigenvalue 4 (theorem 1) and nonpseudogeometric graphs (theorem 2).

Theorem. *Let Γ be a distance-regular graph of diameter $d > 2$ with local subgraphs having parameters from the conclusion theorem 2. If u is the vertex of Γ then one of the following holds:*

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(1) $[u]$ *has parameters* $(27,16,10,8)$, $(63,32,16,16)$, $(135,64,28,32)$, $(189,88,37,44)$, $(243,112,46,56)$, $(279,128,52,64)$ *or* $(351,160,64,80)$ *and* Γ *is Taylor graph;*

(2) $[u]$ *has parameters* $(85,14,3,2)$ *and either* Γ *has intersection array* $\{85,70,1;1,14,85\}$, *or* $\mu\in\{5,7,10\}$, *or* $[u]$ *has parameters* $(169,56,15,20)$ *and* Γ *has intersection array* $\{169,112,1;1,56,169\}$*;*

(3) $[u]$ *has parameters* $(204,28,2,4)$ *and* $\mu\in\{5,6,7,10,12,14,15,17,20,21,25,28,30,35,42,50\}$, *or* $[u]$ *has parameters* $(232,33,2,5)$ *and* $\mu\in\{9,11,12,16,18,22,24,29,33,36,44,48,58,66\}$*;*

(4) $[u]$ *has parameters* $(243,22,1,2)$ *and* $\mu\in\{4,5,6,9,10,11,12,15,18,20,22,27,30,33,36,44,45,54,55,60\}$ *or* $[u]$ *has parameters* $(289,72,11,20)$ *and* Γ *has intersection array* $\{289,216,1;1,72,289\}$*;*

(5) $[u]$ *has parameters* $(325,54,3,10)$ *and* $\mu\in\{45,50,54,65,75,78,90\}$ *or* $[u]$ *has parameters* $(352,26,0,2)$ *and* $\mu\in\{4,5,8,10,11,13,16,20,22,25,26,32,40,44,50,52,55,65,80\}$*;*

(6) $[u]$ *has parameters* $(352,36,0,4)$ *and* $\mu\in\{8,10,11,12,14,15,16,18,20,21,22,24,28,30,32,33,35,36,40,42,44,45,48,55,56,60,63,66,70,72,77,80,84,88,90\}$ *or* $[u]$ *has parameters* $(378,52,1,8)$ *and* $\mu\in\{35,39,42,45,50,54,63,65,70,75,78,90,91,105\}$*;*

(7) $[u]$ *has parameters* $(441,88,7,20)$ *and* Γ *has intersection array* $\{441,352,1;1,88,441\}$ *or* $[u]$ *has parameters* $(505,84,3,16)$ *and* Γ *has intersection array* $\{505,420,1;1,84,505\}$, *or* $[u]$ *has parameters* $(625,104,3,20)$ *and* Γ *has intersection array* $\{625,520,1;1,104,625\}$*;*

(8) $[u]$ *has parameters* $(676,108,2,20)$ *and* $\mu\in\{108,117,126,156,162,169,182,189\}$ *or* $[u]$ *has parameters* $(729,112,1,20)$ *and* $\mu\in\{126,132,154,162,168,189,198,216,231\}$.

References

[1] A. Makhnev, "Strongly regular graphs with nonprincipal eigenvalue 4 and its extensions", *Doklady Mathematics*, Vol. 89, No. 2 (2014).

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