ON KNOTS AND LINKS IN LENS SPACES

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Knot theory is a widespread branch of geometric topology. Although the classical theory regards knots and links in the 3-sphere, the cases where the ambient is a more general (compact) 3-manifolds are widely investigated recently. In this work we focus on the simplest class of closed non-simply connected 3-manifolds: the lens spaces. The study of this kind of knots is relevant for the Berge conjecture [5], in theoretical physics [11] and in biology [2].

Knots/links in lens spaces may be represented with different techniques, as grid diagrams [1], band diagrams [6], mixed link diagrams [8] and other methods. In this work we introduce a representation by regular disk diagrams, generalizing a construction of Drobotukhina [4] regarding the projective space. Namely, consider the lens space $L(p, q)$ as the quotient of the unit ball B^3 by standard identification of the boundary points, we can project any link on the equatorial disk of B^3, so obtaining a regular disk diagram for it. The isotopy equivalence between links in lens spaces can be translated into an equivalence between disk diagrams, via a finite sequence of seven types of moves, generalizing the Reidemeister ones.

Using this representation, a Wirtinger type presentation for the fundamental group, and consequently for the first homology group, of the exterior of the knot/link can be easily obtained. Moreover we deal with the twisted Alexander polynomials of these links, finding different properties and exploiting the connection with the Reidemeister torsion.

Another point of view for the investigation of knots/links in lens spaces is to considering the lift of them in the 3-sphere, under the standard universal covering.

In this way we present an algorithm producing a classical diagram for the lift, starting from a disk diagram of the knot in the lens space. Using this construction we have found different knots and links in $L(p, q)$ with the same lift, showing that the lift is not a complete invariant for knots/links in lens spaces.

The aforementioned results can be found in [10], [3] and [9].

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Joint work with Alessia Cattabriga and Enrico Manfredi.

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