CATEGORIFICATION OF THE COLORED $\mathfrak{sl}_3$-LINK INVARIANT

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The $\mathfrak{sl}_3$ invariant is a link invariant analogous to the Jones polynomial. It has the same definition except that the underlying Lie algebra is $\mathfrak{sl}_3$ instead of $\mathfrak{sl}_2$. Similarly to the Jones polynomial, the $\mathfrak{sl}_3$ invariant admits a colored version for framed links. It is obtained using arbitrary finite dimensional representations of the quantum group $U_q(\mathfrak{sl}_3)$ ($U_q(\mathfrak{sl}_2)$ for the case of the Jones polynomial).

In my talk, I will recall these constructions and explain that one can compute the colored $\mathfrak{sl}_3$ invariant for a framed link K by computing the uncolored version of this invariant on cables of K. The main tools are the Clebsch-Gordan formulas for $U_q(\mathfrak{sl}_3)$: the idea is to express any finite dimensional $U_q(\mathfrak{sl}_3)$-module as a sum (with possibly some signs) of tensor products of the two minuscule representations $V_{0,1}$ and $V_{1,0}$[1]. For example, we have:

$$V_{3,0} \oplus 2V_{1,0} \otimes V_{0,1} \simeq V_{0,0} \oplus V_{1,0}^{\otimes 3},$$

which we may rewrite:

$$V_{3,0} = V_{1,0}^{\otimes 3} \ominus 2V_{1,0} \otimes V_{0,1} \oplus V_{0,0}.$$

The $\mathfrak{sl}_3$ invariant has been categorified by Khovanov in 2003 using webs and foams. It turns out that the Clebsch-Gordan formula can be understood as well at the categorified level: to a module $V_{m,n}$ one can associate a differential graded algebra $A_{m,n}$ which encodes the Clebsch-Gordan formula. The differential graded algebras $A_{m,n}$ are defined via some combinatorial consideration (see figure below).

After shortly recalling the categorification of the $\mathfrak{sl}_3$ invariant, I'll define the graded differential algebras $A_{m,n}$'s, and explain how to categorify the colored $\mathfrak{sl}_3$ invariant.

Note that the same construction has been done for the colored Jones polynomial by Khovanov in 2005.

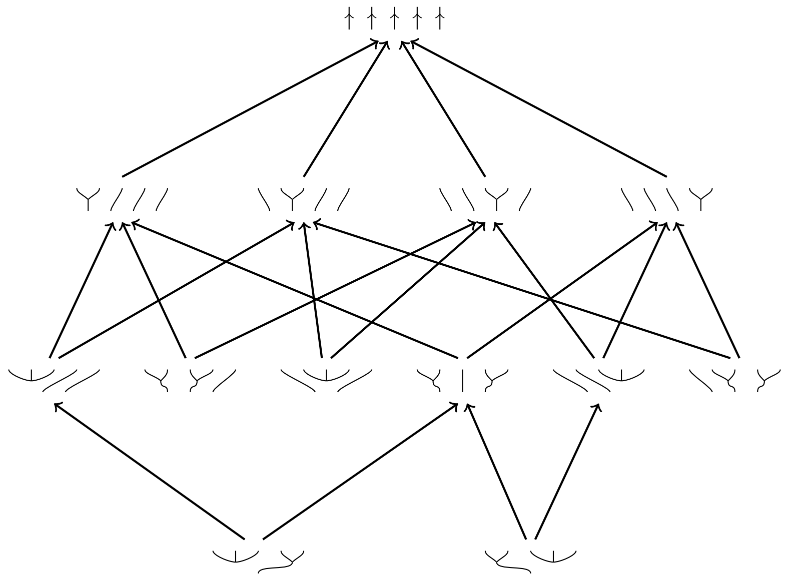

Figure 1. Construction of the differential graded algebra $A_{5,0}$.

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[1] $\mathbb{C} \simeq V_{0,0}$ is seen as the empty tensor product.